

Multidisciplinary Approach to Linear Aerospike Nozzle Design

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A model of a linear aerospike rocket nozzle that consists of coupled aerodynamic and structural analyses has been developed. A nonlinear computational fluid dynamics code is used to calculate the aerodynamic thrust, and a three-dimensional finite element model is used to determine the structural response and weight. The model will be used to demonstrate multidisciplinary design capabilities for relevant engine concepts, assess performance of various multidisciplinary design approaches, and provide a guide for future application development. In this study, the multidisciplinary design problem is formulated using the multidisciplinary feasible strategy. The results for the multidisciplinary feasible formulation are presented with comparisons against sequential aerodynamic and structural designs. Significant improvements are demonstrated by using a multidisciplinary approach in comparison with the single-discipline design strategy.

Nomenclature

A	=	cross-sectional area
$F_{x-\text{thruster}}$	=	axial thruster force
H	=	height of I beam
I_{sp}	=	specific impulse
M	=	Mach number
p	=	static pressure
T	=	axial thrust
W	=	vehicle weight
w	=	I-beam width
x	=	axial coordinate
y	=	vertical coordinate
y_{cowl}	=	cowl height above centerline
γ	=	local ratio of specific heats
θ	=	nozzle surface slope
ϕ	=	thruster angle

Introduction

A MULTIDISCIPLINARY analytic model of a linear aerospike rocket nozzle has been developed; this model includes predictions of nozzle thrust, nozzle weight, and effective vehicle gross liftoff weight (GLOW). The linear aerospike rocket engine is the propulsion system proposed for the X-33 and the VentureStar¹ (Fig. 1) reusable launch vehicle (RLV). The model has been developed to demonstrate multidisciplinary design (MD) capabilities for relevant engine concepts, assess performance of various MD approaches, and provide a guide for future application development. The MD approach is a methodology for the design of complex engineering systems and subsystems that coherently exploits the syn-

ergism of mutually interacting phenomena.² Traditional methods of design, analysis, and optimization have been based on the approach where disciplines are isolated. This work has focused on developing and implementing a baseline MD problem using the multidisciplinary feasible (MDF) strategy.³ This paper presents the results for single-discipline and multidisciplinary designed aerospike rocket nozzles.

The aerospike rocket engine consists of a rocket thruster, cowl, aerospike nozzle, and plug base region (Fig. 2). The aerospike nozzle is a truncated spike (or plug nozzle) that adjusts to the ambient pressure⁴ and potentially integrates well with launch vehicles. The flowfield structure changes dramatically from low altitude to high altitude on the spike surface and in the base region.^{5–7} At high altitudes, the nozzle flow leaving the truncated spike creates a closed wake, called an aerodynamic spike⁸ (giving the aerospike its name). When the wake is closed, the base pressure and the contribution of the base region to the aerospike thrust is independent of altitude. In the early 1960s, aerospike and plug nozzles were the focus of development projects in the United States,⁸ Italy,⁹ and Germany.⁶ More recently, they have been proposed as the propulsion system for the RLV program for NASA¹⁰ and have been studied in the Advanced Rocket Propulsion Technologies¹¹ and Future European Space Transportation Investigations Programme^{12,13} for ESA. This effort is focused on using preliminary design level analysis methods in a multidisciplinary approach to aerospike nozzle design.

The contour of the aerospike nozzle has been traditionally designed by using both simple methods^{14,15} and more elaborate methods based on calculus of variations.^{16–18} These design approaches are adequate for determining an aerodynamic contour that approximates or exactly satisfies a design for maximum thrust at one design condition (usually vacuum). However, the nozzle contour is usually modified as the design of the engine progresses. For example, the length of the nozzle may be varied to improve the thrust-to-weight ratio of the engine. In addition to structural weight effects, the thermal cooling system, propulsion-vehicle integration, thruster contour design,¹⁹ and the fuel-oxidizer delivery system are a few of the topics that are significant in the aerospike nozzle design.

The authors have formed a cooperative research team that initially focuses on multidisciplinary techniques for preliminary design of an aerospike nozzle. In our approach, computational fluid dynamics (CFD) and finite element (FE) codes are used to compute the thrust and the weight, respectively (Fig. 3). Mission-averaged engine-specific impulse and thrust-to-weight ratio are computed and used to determine GLOW from vehicle-based algorithms. The aerospike geometry design parameters (length, base height, and surface contour)

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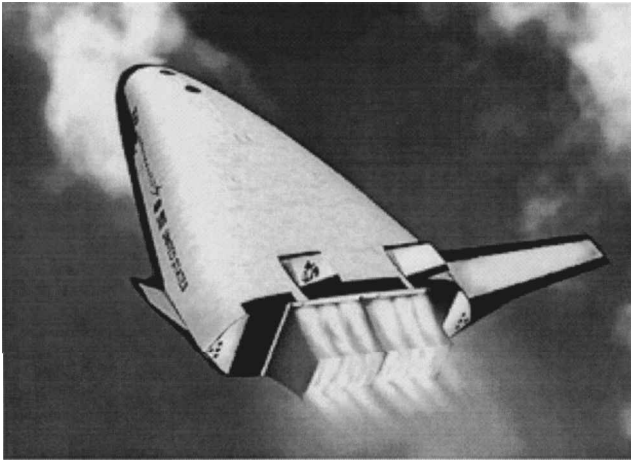


Fig. 1 VentureStar RLV with linear aerospike propulsion system.

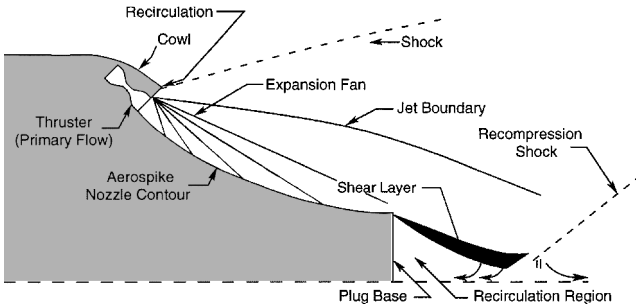


Fig. 2 Aerospike components and flowfield characteristics.

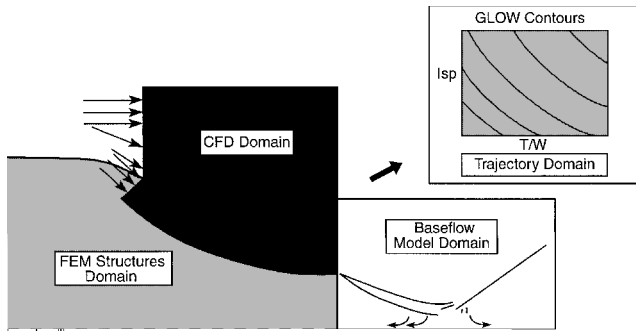


Fig. 3 Multidisciplinary domain decomposition.

and the structural design parameters (thickness, I-beam dimensions, and tube radii) are computed to satisfy structural constraints (displacements, stress, and buckling). An MDF formulation has been implemented to obtain a baseline MD for comparison with future MD formulations. An aerospike design problem has been formulated with a goal of minimizing GLOW. Multidisciplinary synergy has been demonstrated for the optimized design by demonstrating improved performance compared with the more traditional single-discipline design strategy.

The paper is organized in the following manner. In the second section, we first describe the aerodynamic and structural analysis models, along with their corresponding design variables and responses. Subsequently, the multidisciplinary objective function and system responses are discussed. Also, a discussion of the MDF strategy and alternative formulations is included. Results for both designs from single and multidisciplinary problems are presented in the third section. Finally, a summary of this initial work is provided.

Multidisciplinary Problem Development

This effort has concentrated on the development of a multidisciplinary analysis. The analysis includes the use of a nonlinear CFD code and an FE code to calculate aerodynamic thrust and struc-

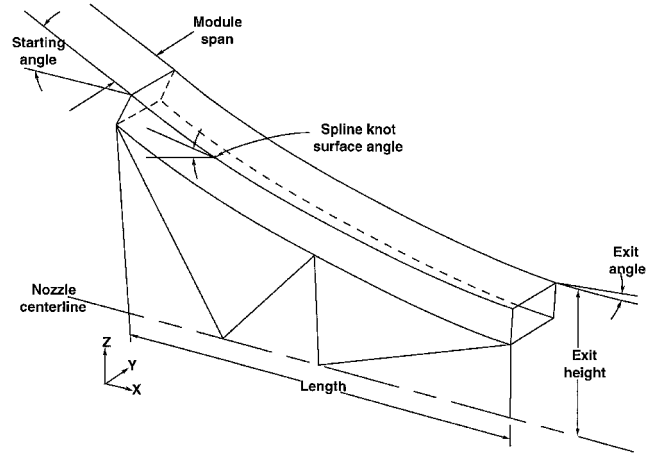


Fig. 4 Aerospike nozzle geometry design parameters.

tural weight. In this section, the discipline problems and their corresponding design parameters are presented, along with the discipline analyses related to the objective function. Next, a baseline MD methodology for the aerospike nozzle design is defined. The design problem is representative of an early preliminary engine design process; it is assumed that a conceptual vehicle design study has been completed that selected a linear aerospike rocket engine as the propulsion system.

Aerospike Nozzle Geometry Parameters

The aerospike nozzle geometry is treated as a two-dimensional surface defined from a fixed point at the end of the cowl (y_{cowl}). The slope of the first point on the aerospike nozzle is set equal to the tangent of the thruster angle. The nozzle surface shape is defined with three spline knots. The thruster exit height is held constant, and the initial nozzle contour point is located in the same axial location as the end of the cowl. The important geometry parameters for an aerospike nozzle are the thruster angle, nozzle surface shape (defined by a cubic spline), nozzle length, and base height (Fig. 4).

Aerodynamic Analysis

The aerodynamic analysis is used to determine data necessary for computing the engine thrust and the static loading on the nozzle structure. For our sample problem, we will use an analysis level that is typically used in initial aerospike nozzle contour design even though our methods can handle more realistic physics. This approach was motivated by the desire to check our analysis methods against legacy design tools and to be able to demonstrate clearly the benefit of our procedures to aerospike nozzle designers. The aerodynamic analysis is computed in three parts: the flow out of the thruster up to the start of the nozzle (one-dimensional analysis), expansion of the flow on the aerospike nozzle (two-dimensional nonlinear CFD), and the base thrust (phenomenological model). In future work, we plan to consider the design of the thruster and the aerospike nozzle simultaneously, which would replace the use of a one-dimensional analysis. However, this approach is efficient and allows direct comparison with existing preliminary aerodynamic analysis and design methods based on the traditional method of characteristics (MOC).

The total axial thrust contribution of the rocket thruster thrust is given by

$$\text{thruster}_{\text{axial}} = pA(1 + \gamma M^2) \cos(\text{thr}_{\text{angle}}) + pA[1/\cos(\text{thr}_{\text{angle}}) - \cos(\text{thr}_{\text{angle}})] \quad (1)$$

where p , A , M , γ , and $\text{thr}_{\text{angle}}$ are static pressure, cross-sectional area, Mach number, ratio of specific heats, and thruster angle (with respect to the horizontal axis). The first term on the right-hand side represents the one-dimensional equation modified by the thruster angle, and the last term quantifies the thrust contribution from the nozzle wall contour between the thruster exit and a vertical line intersecting the cowl. Note that the data are evaluated at the exit plane of the thruster. Downstream of the cowl, the thrust from the

nozzle wall contour is calculated by integrating the nozzle wall pressure over the surface area projected in the axial direction. The base thrust is calculated by multiplying the base pressure by the corresponding base area. The base drag is determined by the engine projected area (width times cowl height) multiplied by the local ambient pressure.

The pressure distribution on the nozzle contour is calculated using a space-marching parabolized Navier–Stokes code.²⁰ The computational domain begins at a vertical plane at the cowl and ends at the nozzle exit. The domain is bounded by the nozzle wall on the bottom and the flow expansion on the top. The combustion products are assumed to be water and to be in vibrational equilibrium. The area ratio of the thruster is large enough such that the gas temperature at the thruster exit is low enough that the chemistry can be considered frozen. The flow that exits the thruster onto the nozzle is assumed to be spatially uniform. The computed flowfield is equivalent to an Euler flowfield solution because the nozzle wall boundary condition imposed was a slip wall condition. The computational grid uses 60 points in the normal direction and approximately 2000 streamwise stations. The nozzle thrust calculation was validated by comparing the solution with an MOC flow solution and performing a grid convergence study using 60, 120, and 240 points in the normal direction. The nozzle thrust calculation takes approximately 15 s on a Sun Sparc Ultra II workstation and differs from the MOC solution by less than 0.08% and from the grid-converged solution by less than 0.1%.

The wall pressure, Mach number, and flow angle at the end of the nozzle are parameters required to analyze the base pressure. The base pressure is computed by using a phenomenological model developed by both Chapman et al.²¹ and Korst et al.²² (for predicting supersonic base flow pressures). This model is believed to be a conservative estimate of the base pressure without bleed and is typical of the level of detail used in a preliminary design.

FE Model and Structure Design Parameters

The structural design concept was generated explicitly for this study and does not relate to a structural configuration that has previously been designed or studied for an aerospike nozzle design. The model represents a typical structural problem that is encountered in the design process. A finite element model (FEM) is generated based on the geometric (Fig. 4) and structural (Fig. 5) design variables. The FEM is a thrust module and begins at the cowl and ends at the base plate. The model is defined using 41 structural-design parameters and additional geometry-design variables. The model has an outer hot wall and an inner cold wall that define top and bottom surfaces of the structural box. The nozzle module is defined using 10 boxes in the longitudinal direction and 4 in the spanwise direction. The sides of the boxes are thin plates (axial and longitudinal webs) with shell stiffeners in the corners of the boxes. The box structure is supported by an I beam that is attached to the cold wall on one side of the thruster module. The web height and flange thickness, in the vertical direction, of the I beam are independent parameters for each box. Six support truss members are connected to the I beam, where each support member is defined by an inner radius and a wall thickness. The FEM is fixed at the attachment point

of the cold wall to the thruster. The support truss members are free to move in-plane; their lower ends remain on the nozzle centerline. The yield strength of the outer hot wall is reduced to account for the temperature dependency of the material properties.

Two FEM codes have been used with different approaches for building the FEM as a function of the geometric and structural design parameters. The first approach used the ANSYS FE code and defined a parametric FEM by using the ANSYS parametric-design language.²³ The second approach used NASTRAN²⁴ for the FE analysis and a code written with MATLAB[®] (Ref. 25) to generate the NASTRAN input as a function of the parametric inputs. Each approach was used successfully, and reasonable agreement between the two analyses was obtained for this sample problem.

The FE solution was obtained for both static and buckling analyses. The FE analysis calculates the weight of the nozzle module. The stresses and resulting displacements from the static analysis are used to partially define the structural constraints, and the buckling analysis is used to calculate the remaining constraints.

GLOW Determination

The objective of most vehicle designs is the minimization of the vehicle weight (either empty or full of fuel). Although this process is normally done when the vehicle is designed, the process is difficult to include in the development of subsystems designs. In this approach, an attempt is made to relate the subsystem design directly to the vehicle performance. Assuming that a performance map for GLOW is developed during the conceptual design, the proposed design strategy is applicable.

To determine the GLOW, the FE weight of the aerospike nozzle is added to the thruster and engine fuel delivery system weight (turbopumps, piping, etc.) to obtain the total engine weight. The thrust and weight predictions are then used to compute engine I_{sp} (specific impulse assuming constant engine mass flow rate) and T/Wt (thrust-to-total-engine-weight ratio) for a near-vacuum condition. A mission-average thrust value is estimated by assuming a 100-s I_{sp} loss at sea level and by assuming that the sea-level operation accounts for 20% of the mission-average thrust. The mission-average I_{sp} and T/Wt values are then used in a table look-up fashion to determine estimates on vehicle GLOW. The GLOW performance map is given in Ref. 26. The main advantage of this approach is that it allows the aerospike design parameters to be determined by an optimization problem defined based on a vehicle-mission objective. Future efforts will include analyses at sea level (important for cooling) and possibly other trajectory points.

MD Methods

MD methods deal with techniques for solving design problems coupled with two or more discipline analyses and constraints. Because of the extreme complexity, problem formulations play a significant role in determining the solution technique and the efficiency of the design improvement (optimization) algorithm. In this investigation, various schemes for MD were investigated, and the MDF approach was selected.

The MDF problem is the optimization of a system objective, subject to satisfying a number of disciplinary analyses and their constraints; at each iteration, a complete multidisciplinary analysis is performed. This is done by iterating between the disciplines until single-discipline feasibility is achieved in all disciplines simultaneously. This iteration process is ordinarily a Gauss–Seidel-like procedure that transfers the output of each discipline into the input of the others, until all discipline output provides solutions to other appropriate disciplines.

A multidisciplinary aerospike-nozzle analysis module was developed to sequence the execution of the discipline codes and to transfer the appropriate input/output data between them (Fig. 6). In the baseline case, aerodynamics and structures are weakly coupled because no feedback is given from structures to aerodynamics. This results in a single aerodynamics-to-structures solution that provides an MDF vector of design variables that is coupled to an optimization program. In the future, when attempting MD of the strongly coupled version of this problem, where structural displacements modify the

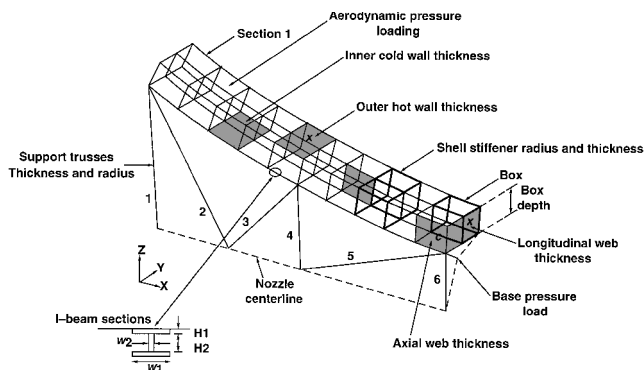


Fig. 5 Aerospike nozzle structural design parameters.

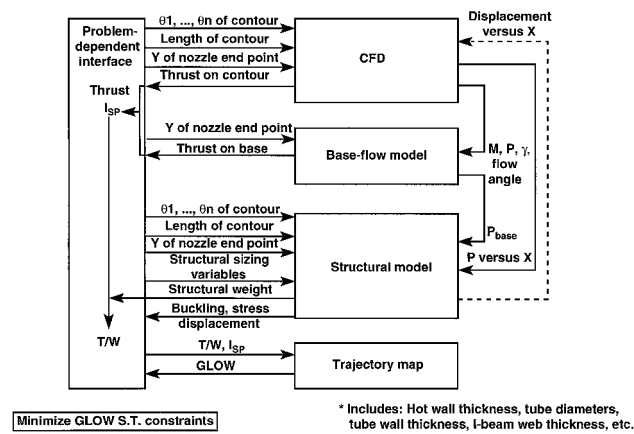


Fig. 6 Multidisciplinary aerospike nozzle analysis.

nozzle shape, we will address the coupling in a number of ways. The promise of other MD formulations is to eliminate the necessity of reaching the expensive multidisciplinary feasibility computation.

Although the MDF approach leads to small, dense optimization problems, the resulting computation is expensive and lengthy because of the necessity to perform not only full multidisciplinary analysis but also sensitivity analysis. The main advantage of the MDF approach is the use of the disciplinary expertise and software. The other important advantage is the availability of an MDF design at each iteration (because it is a design improvement). This is important if the computational expense and time considerations do not allow the optimization procedure to reach completion (minimum objective value). These factors are important from the engineering perspective because the MDF approach is the conventional approach for MD, and this method can serve as a baseline case for future comparisons with computational results of other formulations.

MD Results

In this study, two different methods of design have been investigated. The first method attempts to develop a preliminary design by designing the disciplines sequentially. By the using of the thrust and nozzle weight design solutions obtained, the GLOW is calculated. This first method is a model of a typical design approach. The second method utilizes the MDF formulation and minimizes the GLOW directly subject to satisfying the structural constraints. The gradient-based optimization method, CONMIN,²⁷ was used in all cases. The gradients were calculated using finite differences. The design (optimization) process was required to satisfy the minimum relative or absolute change of the objective function for three consecutive iterations as a stopping criteria.

The design parameters include 4 geometry variables and 14 structural variables (Table 1). The initial geometry-design variables were selected from previous design studies on aerospike nozzles using conventional design methods and are expected to approximate an optimized aerodynamic shape. The number of structural-design variables was reduced by mapping some of the design variables with common attributes into a single-design variable. In particular, the thickness of the I beams was made to be the same in each structural box, and the six structural supports were required to have the same radius and wall thickness. Additional structural parameters are used in forming the constraints (Table 2). The initial values selected for the structural-design parameters resulted in a structural design that was infeasible.

Method 1, Sequential Design (Current Design Practice)

A nozzle geometry design for maximum thrust at the baseline length is presented. Additionally, designs with nozzle lengths of $\pm 10\%$ of the baseline length have also been computed and yield similar results. Four geometry variables were used to determine a nozzle contour for maximum thrust. The convergence of thrust to a maximum value is shown in Fig. 7.

Table 1 Design variables

No.	Type	Description	Initial value, in.
1	Geometry	Thruster angle	—
2	Geometry	Base height	—
3	Geometry	Nozzle wall slope, $x = 10$ in.	—
4	Geometry	Nozzle wall slope (exit)	—
5	Structural	Box depth	7.00
6	Structural	Outer (hot) wall thickness	0.0260
7	Structural	Inner (cold) wall thickness	0.0267
8	Structural	Truss outer radius (1–6)	0.6039
9	Structural	Truss tube thickness (1–6)	0.2864
10	Structural	I-beam column width	1.521
11	Structural	I-beam flange width	0.1935
12	Structural	I-beam flange height (1–10)	0.0711
13	Structural	I-beam column height (1–10)	0.0506
14	Structural	Axial web thickness	0.0242
15	Structural	Longitudinal web thickness	0.0050
16	Structural	Radius of shell stiffener	0.2011
17	Structural	Thickness of shell stiffener	0.0298
18	Structural	Thickness of base plate	0.1665

Table 2 Structural parameters

Parameter name	Value
Hot wall yield stress	16,000 psi
Yield stress except for hot wall	120,000 psi
Safety factor for shell and supports	1.5
Safety factor for buckling	3.0
Maximum vertical displacement of nozzle	0.25 in.

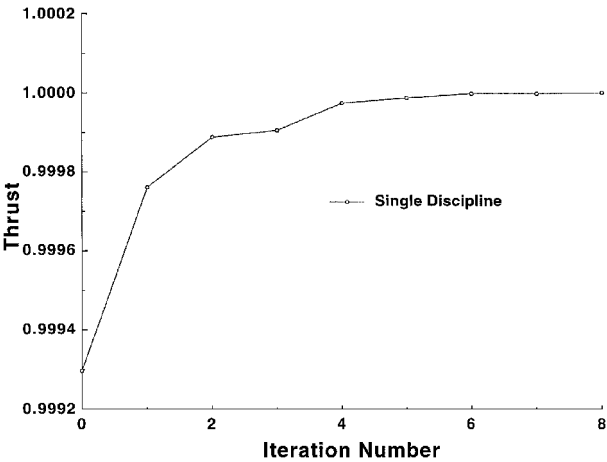


Fig. 7 Convergence of aerospike engine thrust for method 1.

The wall pressure distribution and geometry for the maximized thrust design was input to the structural design. We varied 14 structural-design variables to minimize the weight of the aerospike-nozzle module. The convergence sequence of the nozzle weight is shown in Fig. 8. Note that the nozzle weight at first increases while the optimization strategy adjusts the design for structural feasibility. If an exceptionally small structural thickness is obtained for an optimized design that considers every possible load case, then the structural element may not be needed and could be removed from the design concept. The sequential design results for variable 15, the longitudinal web thickness, converged to a small value (Table 3) for the load cases considered in this paper. Adjusting the design concept to eliminate this design variable was outside the scope of this study.

Method 2, MDF Design

The MDF results are plotted nondimensionalized by the single-discipline result. Method 2 uses the results of method 1 as the initial values of the design variables. The GLOW is minimized for the MD at the baseline length. The improvement in the GLOW for the aerospike nozzle design gives an indication of the relative importance of an MD process. The convergence of the GLOW is

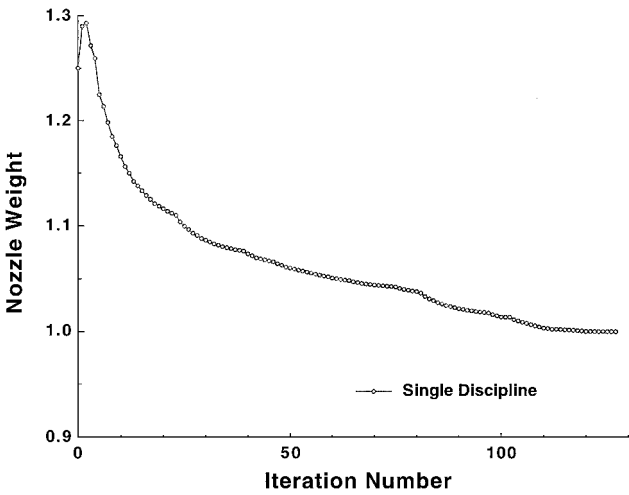


Fig. 8 Convergence of nozzle weight for method 1.

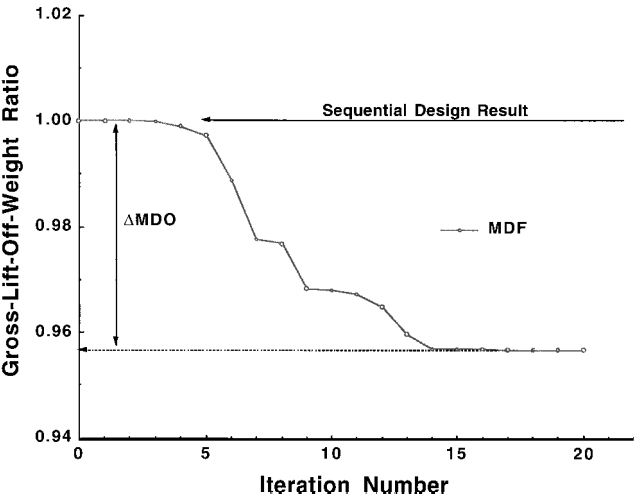


Fig. 9 Convergence of MD objective function, method 2.

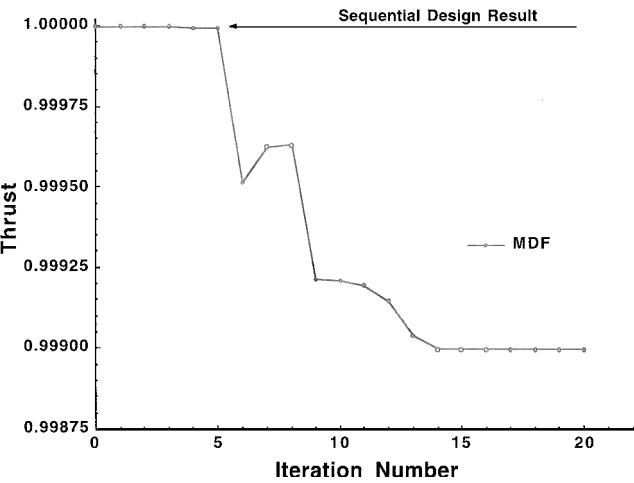


Fig. 10 Convergence of aerospace engine thrust, method 2.

shown on Fig. 9. Greater than 4% improvement, the Δ MDO, in the objective function was obtained with the MD approach. This improvement was obtained by modifying the contour to achieve a multidisciplinary optimum rather than an aerodynamic optimum. The final contour results in lower pressure loading with only minimal loss in thrust (approximately 0.1%, see Fig. 10). The reduced pressure load contributes to a lighter engine structure and, consequently, a 4% reduction in overall vehicle weight.

Table 3 Design variable results

Design variable number	Method 1, single discipline ^a	Method 2, MD ^a
1	0.870	0.843
2	1.101	0.906
3	0.990	1.020
4	0.855	0.863
5	1.294	1.238
6	0.877	0.800
7	0.862	0.791
8	0.971	0.956
9	0.926	0.886
10	0.926	0.880
11	1.000	0.997
12	0.917	0.872
13	1.000	0.998
14	0.763	0.631
15	1.181	1.161
16	0.917	0.869
17	0.926	0.886
18	1.053	0.981

^aResults divided by initial values in Table 1.

Table 4 Design objective function results

Objective	Method 1, single discipline	Method 2, MD
Total thrust ^a	1.000	0.999
Nozzle weight ^a	1.000	0.813
GLOW ^a	1.000	0.957

^aNondimensionalized by optimized design for each discipline (method 1).

The final results for each case and the initial value of the objectives and the design parameters are given in Tables 3 and 4, respectively. Note that significant changes in the design variables are observed between the single-disciplinedesign and the MD with eight variables that vary by more than 5%.

Summary

A multidisciplinary analysis of an aerospike nozzle has been developed both for evaluating MD strategies and new preliminary design processes. Thrust and nozzle wall pressure calculations were made using CFD and were linked to a structural FE analysis for determining nozzle weight and structural integrity. A mission-average specific impulse and engine thrust-to-weight ratio were calculated and used to determine vehicle GLOW (utilizing data defined during the vehicle conceptual design).

The multidisciplinary analysis was integrated with an optimization code that allowed investigation of the MDF strategy. An MD was computed and was compared to the single-disciplinesquential-design approach. The MD resulted in an improvement in the GLOW of greater than 4% over the single-disciplinesquential-designsolution. The improvement was obtained by reducing the nozzle thrust, which resulted in a lower pressure loading on the nozzle structure and a lower nozzle weight. The advantages of the MD approach were demonstrated by the improvement in the design and the ease of including MD variables in the design process.

In the future, our plans are to concentrate in two areas. We are interested in improving the physical model of the aerospike nozzle (by including additional disciplines and additional trajectory points in the multidisciplinary analysis), and we also plan to investigate approximation methods and their use in MD problems.

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